

On the Decidability of Propositional Metric Temporal Calculus *PTC(MT)*

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The paper presents propositional metric temporal calculus *PTC(MT)* dealing with metric properties of time – transitivity and distance between time points. It is shown that *PTC(MT)* is decidable.

1. Introduction

Temporal logic is a kind of symbolic modal logic [1] dealing with domain description statements, which are interpreted over the time flow, either point-based or interval-based. First introduced by Prior in 1957, temporal (or tense) logics relate tenses and modalities, and provide a basis for description of the semantics of the evolving world.

The reviews of known logical systems involving temporal modalities can be found in [2-4]. Among these systems are Lemmon's minimal system K_t (with the unary operators F – “somewhere in the future”, G – “always in the future” and their mirrors), von Wright system “And then” (the binary operator T_w , and basic construct $pT_w q$ – “ p and then q ”), Scott's system “And next instant” (the unary operator T_s , basic construct $T_s p$ – “in the next time point will be p ”). Temporal aspect is also of great interest for hybrid logics, where it is possible to directly refer to worlds/times/states in logical formulae. E.g. Rescher's chronological calculus [5] introduces the operator of chronological realization, which binds an event to the particular real date/time.

Temporal logics are widely accepted languages for specifying properties of reactive systems and their behaviour over time [6-7], and for the description of concurrent object-based systems: process controls, fault tolerant systems, distributed AI [8]. Its application to the description of evolving behaviour of dynamic domains is under detailed investigation, particularly for the purposes of knowledge representation on the Semantic Web (see e.g. [9-10]).

The examples of propositional temporal logics for linear time are *LTL* [11], *PTL* [12], *Timed PTL* [7], and the set of Propositional Linear Temporal Logics from [4].

Metric temporal logic with modalities F_n (“it will be the case after n time points”) and P_n (“it was the case n time points ago”) allows in addition to description of precedence of events to explicitly state distances (in time points) between occurrence of events.

This logic is positioned between non-metrical temporal logics and hybrid logics. Indeed, from the one point, constructs like $F_n p \vee F_m q$ tell that p will be true in n time

moments or q will be true in m time moments, thus one can use primitive arithmetic operations to calculate the difference between time moments when the p or q occur. From the other point, it is impossible to set or get absolute values of time moments when p or q occur.

The paper describes propositional metric temporal calculus $PTC(MT)$, and presents results of decidability analysis of $PTC(MT)$.

The paper is structured as follows: Section 2 introduces the $PTC(MT)$; Section 3 describes the tableau procedure for checking formula satisfiability; Section 4 analyses decidability of $PTC(MT)$; Section 5 concludes the paper.

2 $PTC(MT)$

Propositional metric temporal calculus considers time having linear discrete structure, infinite into the past and to the future, assumes that time points are organized with reflexive and transitive ordering relation.

Such structure of time is isomorphic to the structure $\langle Z, < \rangle$, where Z - is a set of integers, and $<$ - is a strict ordering relation.

2.1 Alphabet, formulae, axioms, deduction rules

The alphabet of $PTC(MT)$ consists of:

- (a) Propositional variables p, q, r, s, \dots ;
- (b) Primitive propositional connectives \neg, \supset , and additional connectives \wedge, \vee, \equiv , defined over primitive ones in the usual way;
- (c) Temporal operators F_n, P_n (F_n - «it will be the case after n time points», P_n - «it was the case n time points ago»);

$PTC(MT)$ terms are:

- (a) ν, ν_1, ν_2, \dots are natural numbers and «0»;
- (b) $i, i_1, \dots, j, j_1, \dots$ are numerical variables;
- (c) if n_1, \dots, n_m are natural numbers and «0» or numerical variables, and θ - m -ary operator, then $\theta(n_1, \dots, n_m)$ - is a term.

Formulae are constructed following the rules:

- (a) Every propositional variable is a formula;
- (b) If φ and ψ are formulae, then $\neg\varphi, \varphi \supset \psi, \varphi \wedge \psi, \varphi \vee \psi, \varphi \equiv \psi$ are also formulae;
- (c) If Pr^m is a predicate letter denoting m -ary predicate, defined over integers (e.g., « \Rightarrow », « $\langle \rangle$ », ...), and n_1, \dots, n_m - are terms, then $\text{Pr}^m(n_1, \dots, n_m)$ - is a formula;
- (d) If φ - is a formula, then $F_n\varphi, P_n\varphi, \exists i\varphi, \forall i\varphi$ - are also formulae.

Alphabet of $PTC(MT)$ is defined.

Definition 1.

Numerical variable i occurs free in a formula φ , if it is not within the scope of any quantifier in φ .

Definition 2.

Term n is free in a formula φ for a numerical variable j , if there are no free occurrences of j in φ , such that j is within the scope of any quantifier $\forall i_m$, where i_m is a numerical variable in the term n .

$PTC(MT)$ axioms set consists of all axioms of the propositional calculus and some axioms of temporal logic, taken from [1-3].

Following formulae are axioms (propositional axioms are correspondent to L4 system, see [13, p.49]):

- (A1) $p \supset (q \supset p)$;
- (A2) $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$;
- (A3) $p \wedge q \supset p$
- (A4) $p \wedge q \supset q$
- (A5) $p \supset (p \vee q)$
- (A6) $q \supset (p \vee q)$
- (A7) $p \supset (q \supset (p \wedge q))$
- (A8) $(p \supset q) \supset ((r \supset q) \supset ((p \vee r) \supset q))$
- (A9) $(p \supset q) \supset ((p \supset \neg q) \supset \neg p)$
- (A10) $\neg\neg p \supset p$
- (AMT1) $(\neg F n \neg (p \supset q)) \supset (F n p \supset F n q)$ – logical homogeneity in the future
- (AMT1.1) $(\neg P n \neg (p \supset q)) \supset (P n p \supset P n q)$ – logical homogeneity in the past
- (AMT2) $F n \neg P n \neg p \supset p$
- (AMT2.1) $P n \neg F n \neg p \supset p$
- (AMT3) $F m \exists i F i p \supset \exists i F m F i p$
- (AMT3.1) $P m \exists i P i p \supset \exists i P m P i p$
- (AMT4) $F m \exists i P i p \supset \exists i F m P i p$
- (AMT4.1) $P m \exists i F i p \supset \exists i P m F i p$
- (AMT5) $F(m+n)p \supset F m F n p$
- (AMT5.1) $P(m+n)p \supset P m P n p$
- (AMT6) $\neg F n p \supset F n \neg p$ – infinity into the future
- (AMT6.1) $\neg P n p \supset P n \neg p$ – infinity into the past
- (AMT7) $F n \neg p \supset \neg F n p$ – nonbranching in the future
- (AMT7.1) $P n \neg p \supset \neg P n p$ – nonbranching in the past
- (AMT8) $F m F n p \supset F(m+n)p$ – transitivity in the future
- (AMT8.1) $P m P n p \supset P(m+n)p$ – transitivity in the past
- (AMT9) $(m = n + k) \supset (F m P n p \supset F k p)$ – iteration of temporal modalities

Propositional axioms are independent with respect to $PTC(MT)$, the same applies for temporal axioms.

Deduction rules for calculus $PTC(MT)$ are:

- (R1) $\frac{\varphi, \varphi \supset \psi}{\psi}$ – Modus Ponens

(R2) $\frac{\varphi(p)}{\psi(p/\gamma)}$ – substitution rule (ψ is obtained after replacing in φ all

occurrences of a propositional variable p with formula γ)

(R3) $\frac{\varphi}{\neg F\neg\varphi}$ – the rule of deriving “always in the future”

(R4) $\frac{\varphi}{\neg P\neg\varphi}$ – the rule of deriving “always in the past”

Let φ be a $PTC(MT)$ formula that does not contain numerical variable i , $\varphi[j/i]$ be a $PTC(MT)$ formula with all free occurrences of a numerical variable j replaced with i . Then the following deduction rule may be applied:

(R5) $\frac{\varphi[j/i]}{\forall i\varphi}$ – the generalization rule

If φ is a $PTC(MT)$ formula which contains numerical variable i , and $\varphi[i/n]$ be a $PTC(MT)$ formula with all occurrences of a numerical variable i replaced with term n , which is free for i in φ , then the following deduction rule may be applied:

(R6) $\frac{\forall i\varphi}{\varphi[i/n]}$

Calculus is constructed.

Throughout this paper we restrict the discussion with binary operations “+”, “-” for $PTC(MT)$ terms construction and use the only binary predicate “=” (“equality”).

Definition 3.

Formula φ is called **atomic**, if φ is either a propositional variable or its negation, or a formula of the view $\text{Pr}^m(n_1, \dots, n_m)$ or its negation.

Definition 4.

Formula φ is in **negation normal form** (n.n.f.), if for every subformula $\neg\psi$ formula ψ is atomic, and the whole formula φ is constructed without binary propositional connectives \supset, \equiv .

Theorem 1.

Let φ be a formula from $PTC(MT)$.

Then $\vdash\text{---}\varphi \equiv \psi$, where ψ - is a formula in negation normal form (n.n.f.).

The proof of this fact is shown in the [14].

Definition 5.

Formula φ is in **$FnPn$ -normal form** ($FnPn$ -n.f.), if it can be presented as:

$$\begin{aligned}
 \varphi \equiv & \bigvee_{k=1}^N \left(\bigwedge_{r^1=0}^{N_k^1} F V_{kr^1} \varphi_{kr^1} \wedge \bigwedge_{r^2=0}^{N_k^2} P V_{kr^2} \varphi_{kr^2} \wedge \right. \\
 & \bigwedge_{r^3=0}^{N_k^3} \bigwedge_{r^4=0}^{N_k^4} \exists i_{kr^3} \text{Pr}^{s+1}(i_{kr^3}, v_1, \dots, v_s) \wedge \forall i_{kr^4} \text{Pr}^{s+1}(i_{kr^4}, v_1, \dots, v_s) \wedge \\
 & \left. \bigwedge_{\substack{\alpha, \beta \\ r^j=0, N_k^j \\ d=0, D_k^j}} \alpha V_{kr^j} \beta i_{kr^j_1} \alpha i_{kr^j_1} \dots \beta i_{kr^j_d} \alpha i_{kr^j_d} \varphi_{kr^j} \right)
 \end{aligned}$$

where

- N - is a number of disjuncts in a formula,
- k - is an internal index for referencing disjuncts within the formula,
- $N_k^j \geq 0$ - is a number of conjuncts of a particular conjunct form within k -th disjunct
- $j = 1, \dots, 2^{2-D_k}$ - is an index of a particular conjunct form within k -th disjunct,
- $\alpha \in \{F, P\}$ - is a symbol, partially denoting one of temporal modalities,
- $\beta \in \{\exists, \forall\}$ - is a symbol denoting one of quantifiers,
- $r^j = 0, \dots, N_k^j$ - is an internal index for referencing formulae of a particular conjunct form within k -th disjunct,
- $d = 1, \dots, D_k^j$ - is an internal index for referencing elements of the form $\beta_{kr^j} i_{kr^j} \alpha_{kr^j} i_{kr^j}$ within a formula in the r^j -th conjunct of the particular conjunct form within k -th disjunct,
- $D_k^j \leq D_k$ - is the number of quantifiers in the particular conjunct form within k -th disjunct,
- D_k - is the maximal number of quantifiers among all particular conjunct forms within k -th disjunct,
- φ_{kr^j} - are atomic formulae.

$FnPn$ -n.f. of a $PTC(MT)$ formula is a list of alternative histories of states of some object from a domain.

Theorem 2.

Let φ be a formula of $PTC(MT)$ in n.n.f. Then $\vdash \varphi \equiv \psi$, where ψ is a formula in $FnPn$ -normal form. The proof of this fact is shown in the [14].

3 Tableau procedure for checking $PTC(MT)$ formula satisfiability

Construct a model of an arbitrary $PTC(MT)$ formula. It is wide accepted technique to use tableau rules to construct a model for a modal system.

Definition 6.

Let φ be a formula in $FnPn$ -n.f., and ψ be a subformula of φ . A sequence of formulae lists $\langle \zeta_0, \zeta_1, \dots, \zeta_m, \zeta_{-1}, \dots, \zeta_{-s} \rangle$, linearly ordered with a binary relation R

(reflexive and transitive), forms a **chain** Z_φ for the formula φ , if this sequence is constructed following the set of rules, presented in the Table 1.

Table 1. Rules for construction of a semantic tableau for checking $PTC(MT)$ formula satisfiability.

| | |
|---------------------|---|
| (0-rule) | Condition: $\psi = \varphi$ |
| | Action: $\zeta_0 = \psi$ |
| $(\wedge$ -rule) | Condition: 1. $\psi = \psi_1 \wedge \psi_2$ 2. $\{\psi_1, \psi_2\} \cap \zeta = \emptyset$ |
| | Action: $\zeta = \zeta \cup \{\psi_1, \psi_2\}$ |
| $(\vee$ -rule) | Condition: 1. $\psi = \psi_1 \vee \psi_2$ 2. $\{\psi_1, \psi_2\} \cap \zeta = \emptyset$ |
| | Action: Either $\zeta = \zeta \cup \{\psi_1\}$ or $\zeta = \zeta \cup \{\psi_2\}$ |
| $(Fv$ -rule) | Condition: 1. $\psi = Fv\psi_1$ 2. $\psi \in \zeta_k$ 3. $v \geq 1$ |
| | Action: 1. If there is no $\zeta_{k+1} : \zeta_{k+1} \in Z_\varphi$, then such list is created and new formula $\psi' = F(v-1)\psi_1$ is added to the ζ_{k+1} , $\psi' \in \zeta_{k+1}$ 2. If exists $\zeta_{k+1} : \zeta_{k+1} \in Z_\varphi$, then $\psi' = F(v-1)\psi_1$ is added to the ζ_{k+1} , $\psi' \in \zeta_{k+1}$ 3. Between ζ_k and ζ_{k+1} relation $R(\zeta_k, \zeta_{k+1})$ is set. |
| $(Pv$ -rule) | Condition: 1. $\psi = Pv\psi_1$ 2. $\psi \in \zeta_k$ 3. $v \geq 1$ |
| | Action: 1. If there is no $\zeta_{k-1} : \zeta_{k-1} \in Z_\varphi$, then such list is created and new formula $\psi' = P(v-1)\psi_1$ is added to the ζ_{k-1} , $\psi' \in \zeta_{k-1}$ 2. If exists $\zeta_{k-1} : \zeta_{k-1} \in Z_\varphi$, then $\psi' = P(v-1)\psi_1$ is added to the ζ_{k-1} , $\psi' \in \zeta_{k-1}$ 3. Between ζ_k and ζ_{k-1} relation $R(\zeta_k, \zeta_{k-1})$ is set. |
| $\exists iFi$ -rule | Condition 1. $\psi = \exists iFi\psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$ |

| | | |
|--|-----------|---|
| | Action | Either $\psi_1 \in \zeta_k$ or new formula $\psi' = F1 \exists iFi \psi_1$ belongs to ζ_k , $\psi' \in \zeta_k$ |
| $\exists iPi$ -rule | Condition | 1. $\psi = \exists iPi \psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$ |
| | Action | Either $\psi_1 \in \zeta_k$ or new formula $\psi' = P1 \exists iPi \psi_1$ belongs to ζ_k , $\psi' \in \zeta_k$ |
| $\forall iFi$ -rule | Condition | 1. $\psi = \forall iFi \psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$ |
| | Action | 1. $\psi_1 \in \zeta_k$ 2. For each $\zeta_j : \zeta_j \in Z_\varphi, j > k$, such that the relation $R(\zeta_k, \zeta_j)$ is set, $\psi \in \zeta_j$ |
| $\forall iPi$ -rule | Condition | 1. $\psi = \forall iPi \psi_1$ 2. $\psi \in \zeta_k, \psi_1 \notin \zeta_k$ |
| | Action | 1. $\psi_1 \in \zeta_k$ 2. For each $\zeta_j : \zeta_j \in Z_\varphi, j < k$, such that the relation $R(\zeta_k, \zeta_j)$ is set, $\psi \in \zeta_j$ |
| $\exists iPr^2(i, \theta(v_1, v_2))$ -rule (for predicate letter “=”) | Condition | 1. $\psi = \exists iPr^2(i, \theta(v_1, v_2))$ 2. $\psi \in \zeta_k$ |
| | Action | If there is no $\zeta_i \in Z_\varphi$, such that $i = \theta(v_1, v_2)$, then such list is created. |
| $\forall iPr^2(i, \theta(v_1, v_2))$ -rule (for predicate letter “=”) | Condition | 1. $\psi = \forall iPr^2(i, \theta(v_1, v_2))$ 2. $\psi \in \zeta_k$ |
| | Action | If there is no $\zeta_i \in Z_\varphi$, such that $i = \theta(v_1, v_2)$, then such list is created. |

Table 1 does not contain rules for resolving formulae like $\psi = Fi\psi_1$, where i is a numerical variable, or like $\psi = \forall i\psi_1$. Such formulae can be presented in the form $\forall jFj\psi_1$ with application of the deduction rules R5, R6.

It also should be pointed out that the $\forall iFi$ - and $\forall iPi$ -rules reflect the transitivity and reflexivity of the relation R between possible worlds at different time points. According to the definition of a model for a modal system (see [15]) the model of the propositional metric temporal calculus $PTC(MT)$, constructed according to the rules from Table 1, is S4-model.

Definition 7.

A set $\{Z_\varphi^1, \dots, Z_\varphi^k\}$ of chains constructed according to the rules enlisted in the Table 1, is called a **construction** C_φ .

Definition 8.

Chain Z_φ is **closed**, if it contains a formulae list ζ such that for some propositional variable p both p and $\neg p$ are in ζ . Construction C_φ is **closed** if all chains in it are closed.

Given a $PTC(MT)$ formula φ , its construction creation procedure can be described as follows. Construction creation starts from applying 0-rule, then apply \vee -rule until there will not be any unresolved subformulae ψ having disjunction, then apply \wedge -rule until there will not be any unresolved subformulae ψ having conjunction. After that apply $\exists iPr^2(i, \theta(v_1, v_2))$ - and $\forall iPr^2(i, \theta(v_1, v_2))$ -rules, which will introduce new (though empty) formulae lists, then apply Fv - and Pv -rules until there will not be any unresolved subformulae of that form. Finally, apply $\exists iFi$ -, $\exists iPi$ -rule and then $\forall iFi$ - and $\forall iPi$ -rules. This process will be continued until for each chain there will be a formulae list, which fulfills one of the following two conditions: either this chain is closed, or this chain with the same set of formulae is already in the construction.

Definition 9.

Construction C_φ is **complete** if no tableau rule is applicable to it.

Definition 10.

Let φ be a formula of $PTC(MT)$. A **model for** φ will be any chain Z_φ , which is not closed.

Definition 11.

Formula φ is **satisfiable** if and only if φ has a model defined over the construction C_φ .

Definition 12.

Formula φ is **logically valid** (denoted as \models) if and only if $\neg\varphi$ does not have a model defined over the construction $C_{\neg\varphi}$ (in other words, $\neg\varphi$ is **unsatisfiable**).

Metatheorem 1.

$PTC(MT)$ is sound.

The proof of this fact is shown in the [14].

Metatheorem 2.

$\vdash\varphi$ iff $\models\varphi$ (completeness of $PTC(MT)$)

The proof of this fact is also shown in the [14].

4 $PTC(MT)$ decidability

A formal theory is decidable if there is an effective decision procedure of checking whether a given formula is satisfiable.

Let $|C_\varphi|$ be cardinality of a construction C_φ of a formula φ – the number of chains Z_φ for the formula φ . Let $|Z_\varphi|$ be cardinality of a chain Z_φ – the number of formulae lists ζ_i in the chain. Finally, let $|\zeta|$ be cardinality of a formulae list ζ from the chain Z_φ of the construction C_φ of formula φ – the number of formulae in the formulae list ζ .

Lemma 1.

For arbitrary $PTC(MT)$ formula φ the process of completing a construction C_φ always terminates after finitely many steps.

Proof: according to the Theorem 2 without loss of generality let φ be in $FnPn$ -n.f. Construction C_φ is finite if and only if it consists of finite set of chains Z_φ , each chain Z_φ is also finite, i.e. consists of finite set of formulae lists ζ_i , and each formulae list ζ_i also consists of finite set of subformulae of the formula φ .

The analysis of the rules from Table 1 shows that

$$\forall Z_\varphi \in C_\varphi \quad |Z_\varphi| := |Z_\varphi| + 1 \text{ in case of application of } F\nu -, P\nu -, \exists iFi -, \exists iPi ,$$

$\exists iPr^2(i, \theta(\nu_1, \nu_2)) -, \forall iPr^2(i, \theta(\nu_1, \nu_2)) -$ rules, and remains the same otherwise.

Consider a chain Z_φ and evaluate the number of formulae in a given formulae list:

$$\forall \zeta_i \in Z_\varphi \quad |\zeta_i| := |\zeta_i| + 1 \text{ in case of application of } F\nu -, P\nu -, \exists iFi -, \exists iPi -, \forall iFi -, \forall iPi -, \vee - \text{ rules,}$$

$\forall \zeta_i \in Z_\varphi \quad |\zeta_i| := |\zeta_i| + 2$ in case of application of \wedge -rule, and remains the same otherwise.

Recall that there are no more than $2^{2 \cdot D_k}$ different conjunct forms in the k -th disjunct in φ , and no more than N_k^j conjuncts within each conjunct form. The cardinality of the initial formulae list, ζ_0 , for the formula φ is bounded:

$$|\zeta_0| \leq 2^{N_k^1 + N_k^2 + \dots + N_k^{2 \cdot D_k}} - 1$$

It is obvious, that $|\zeta_i| \leq |\zeta_0|$ for any $\zeta_i \in Z_\varphi$, as far as only in ζ_0 will be conjuncts of the forms $\exists i_{kr^3} Pr^{s+1}(i_{kr^3}, \nu_1, \dots, \nu_s)$, $\forall i_{kr^3} Pr^{s+1}(i_{kr^3}, \nu_1, \dots, \nu_s)$.

The cardinality of the chain, corresponding to the whole formula φ is also bounded:

$$|Z_\varphi| \leq \max\{\nu : F\nu\varphi_{kr^j} \in Sub(\varphi), F\nu\beta i_{kr^j_1} \alpha i_{kr^j_1} \dots \beta i_{kr^j_d} \alpha i_{kr^j_d} \varphi_{kr^j} \in Sub(\varphi)\} + \\ + \max\{\nu : P\nu\varphi_{kr^j} \in Sub(\varphi), P\nu\beta i_{kr^j_1} \alpha i_{kr^j_1} \dots \beta i_{kr^j_d} \alpha i_{kr^j_d} \varphi_{kr^j} \in Sub(\varphi)\}$$

where:

- φ_{kr^j} - are atomic formulae within φ ,
- $Sub(\varphi)$ - is the set of all subformulae of φ .

Among all possible chains Z_φ for φ (the number of all possible chains is restricted with the cardinality of the $Sub(\varphi)$) choose the chain with maximal cardinality, Z_φ^{\max} .

The cardinality of the construction C_φ for the formula φ can be restricted as $|C_\varphi| \leq |Z_\varphi^{\max}| \cdot |\zeta_0|$. End of proof.

Metatheorem 3.

$PTC(MT)$ is decidable.

The proof of this fact is based on Lemma 1.

5 Conclusions

The paper reports on the results of evaluation of the propositional metric temporal calculus $PTC(MT)$. It was shown that $PTC(MT)$ with temporal modalities F_n and P_n is decidable.

The work will be continued in the following direction: all results obtained for the propositional metric temporal system $PTC(MT)$ will be considered for the Description Logics family, which are de facto standard for presentation of ontologies on the Semantic Web.

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