HYBRID PROPOSITIONAL METRIC TEMPORAL CALCULUS

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Hybrid logics (see introduction in [Bl00]) form a class of modal logics that allows referring to elements of Kripke models of modal formula – "possible worlds" - directly in formulae.

Logical languages used for hybrid logic are at least two-sorted: they have a special sort of atomic objects called *nominals*. Each nominal names exactly one possible world of Kripke model. Depending on the basic modal logic, possible worlds may represent moments of time, states, locations etc.

Hybrid temporal logics allow description of complex system, where temporal constraints and explicit time points referencing are embedded into the theory describing the behavior of a complex system. However, little attention was paid to the representation of metric properties of time (e.g. distance) in hybrid temporal logic.

Investigation of properties of hybrid propositional metric temporal calculus *HPTC*(*MT*) has led to the following results:

Definition 1. The alphabet of *HTPC*(*MT*).

Let $PROP = \{p, q, p', q', ...\}$ be propositional symbols, $NOM = \{a, b, c, ...\}$ be non-empty set of nominals, $NUM = \{n, m, k, ...\}$ be a set of natural numbers and number "zero", $N \cup \{0\}$, $NUMVAR = \{i, j, k, ...\}$ be a set of numerical variables, $NUM \cup NUMVAR = \{x, x_1, x_2, ...\}$ be a set of names, defining numerical variables and the elements of *NUM*, $PRED = \{\Pr^s, s = 1,2,...\}$ be a set of *s*-ary predicate symbols, defined over $N \cup \{0\}$, $MOD = \{Fx, Px, x \in NUM \cup NUMVAR\}$ be a set of modalities. Then well-formed formulae set *WFF* on symbols from *NOM*, *PROP*, *NUM*, *NUMVAR*, *PRED* and *MOD* consists of the following formulae:

$$WFF := a \mid p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \supset \psi \mid \Pr^s(x_1, ..., x_s) \mid Fx\varphi \mid Px\varphi \mid \exists i\varphi \mid \forall i\varphi \mid @_a\varphi,$$

for each $\varphi \in WFF$.

Definition 2. The model of *HTPC*(*MT*).

HPTC(MT) is interpreted over Kripke frame [Bl00] for linear unbounded time with some metric:

$$\mathrm{M} = < W, dist, \{R_F, R_P\}, \xi, \xi^s_{PRED} > .$$

Here *W* is a non-empty set of worlds, $dist: W \times W \to N \cup \{0\}$ - is the metric on W, R_F, R_P are binary accessibility relations on *W*, respectively, to the future and to the past, $\xi : PROP \cup NOM \to 2^W$ is an interpretation function, and for each nominal a $\xi(a) \in W$ - a single world, corresponding to the nominal a, $\xi^s_{PRED} : PRED \to 2^{W^s}$ is an interpretation function for predicate symbols.

As far as linear unbounded time structure is isomorphic to the set of integers, Z, without loss of generality it will be assumed the model to be

$$\mathrm{M} = < Z, dist, \{R_F, R_P\}, \xi, \xi^s_{PRED} > ,$$

where *dist* is Euclidean metric.

For the hybrid propositional metric temporal calculus *HPTC(MT)* defined is the negative normal form (n.n.f.) of a *HPTC(MT)* formulae. Proved is the existence of n.n.f. for arbitrary *HPTC(MT)* formulae. Extension of Beth's semantic tableaux method [Kr74] introduced for (non-hybrid) propositional metric temporal calculus *PTC(MT)* in [Ke06] is used to prove completeness, soundness and decidability of *HPTC(MT)*. The computational complexity of the *HPTC*(*MT*) is shown to be EXPTIME.

References

[Kr74] Kripke S.A. A completeness theorem in modal logic. In (Feys R.) Modal Logic. M.: Nauka, – 1974. – 520 p. (in Russian)

[Ke06] Keberle N.G. Properties of Propositional Metric Temporal Calculus for Description of Evolving Conceptualization. In: Problems of Applied Mathematics and Mathematical Modelling. Dniepropetrovks: DNU Press, – 2006. – pp.80-100.

[Bl00] Blackburn P. Representation, Reasoning, and Relational Structures: a Hybrid Logic Manifesto. Logic Journal of IGPL, 8(3): 339-625, 2000.